

Message Authentication Code (MAC) & Chosen-Ciphertext Attack

$CCAExp_A^\epsilon(n)$

$K \leftarrow G(1^n)$
 $(m_0, m_1) \leftarrow A(1^n)$
 $b \leftarrow \{0, 1\}$
 $c = E(K, m_b)$
 $\hat{b} \leftarrow A(1^n, c)$
 if $b = \hat{b}$ return 1
 else return 0

$E = (G, E, D)$ is a private-key encryption scheme
 A is a PPT adversary

E is CCA-secure if $\forall A, PPT$

$$\text{Prob}[CCAExp_A^\epsilon(n) = 1] \leq \frac{1}{2} + \nu(n)$$

cannot ask for decryption of c

Obs. The CPA-secure encryption scheme based on PRF obtained in Block 3 is not CCA secure. Why? Exercise for students

This block

PRF \rightarrow MAC \rightarrow CCA

\uparrow (Block 3)
 \downarrow (Block 2) \leftarrow trivial
 MAC \leftarrow CPA

$(KeyGen, Mac, Vrfy) = MAC$
 $KeyGen \rightarrow Key$
 $Mac : Message + Key \rightarrow Tag$
 $Vrfy : Message + Key + Tag \rightarrow \{0, 1\}$

$MACExp_A^{Mac}(n)$

$k \leftarrow KeyGen(1^n)$
 $(m, t) \leftarrow A^{Mac(\cdot, k)}(1^n)$ (set of queries)
 if $Vrfy(m, k, t) = 1 \wedge (m \notin Q)$
 then return 1

MAC unforgeable if $\forall PPTA$

$$\text{Prob}[MACExp_A^{Mac}(n) = 1] \leq \nu(n)$$

PRF \rightarrow MAC

Let \mathcal{F} be a family of PRF
 Mac for messages of length n

KeyGen(1^n)
 $k \leftarrow \{0,1\}^k$
 return k

Mac(m, k)
 return $t = f_k(m)$

Verify(m, k, t)
 iff $t = f_k(m)$

Thm If \mathcal{F} is a PRF then Mac is unforgeable
Pf.

Consider RMacExp that is the experiment in which A 's queries are answered not by running Mac(m, k) but by running modified Mac in which we use a truly random function instead of f_k .

Prob[RMacExp(1^n)=1] $\leq 2^{-n}$. If $\exists A$ s.t. Prob[MacExp(1^n)=1] $\geq \frac{1}{\text{poly}(n)}$
 then we can construct D that distinguishes PRF from RF

$D^O(1^n)$

- run $A(1^n)$ with $Q = \emptyset$
 query m from A
 return $O(m)$ and store m in Q ($Q \leftarrow Q \cup \{m\}$)
- let (m, t) be the output of A
- if $t = O(m)$ and $m \notin Q$ return 1
 else return 0

If O is a PRF then A 's view is like in MacExp \Rightarrow
 Prob[$D^O = 1$] = Prob[MacExp(1^n)=1] $\geq \frac{1}{\text{poly}(n)}$

If O is a RF then A 's view is like in RMacExp \Rightarrow
 Prob[$D^O = 1$] = Prob[RMacExp(1^n)=1] = 2^{-n}

$\Rightarrow D$ break the pseudorandomness of \mathcal{F} .